Non-Linear Control Strategy for Type-I Diabetes Mellitus Patient with Luenberger Observer

[1]Rohit Sharma, [2]Prof. Nitin Bhitre
[1]Sardar Patel College of Engineering
Email:rrohit.ssharma96@gmail.com
[2]Sardar Patel College of Engineering
Email:n_bhitre@spce.ac.in

Abstract

Luenberger observer is designed to estimate the states of the Bergman Minimal model of Glucose-insulin to continuously control the blood glucose level of Type-1 diabetic mellitus patient as close as basal value. The controller is designed by philosophy of nonlinear Backstepping method. Observer is designed by considering the linear model of the Bergman minimal model. MATLAB simulation shows the effectiveness of the observer in controlling blood glucose level of the realistic patient.

Keywords: Diabetes; Bergman model; Basal value; Controller; Luenberger observer; Linearization;

1.Introduction

WHO reports shows that 1.5 million deaths caused by Diabetes in 2012. Additional 2.2 million deaths caused by higher than optimal blood glucose. 43% of these 3.7million deaths occurs before the age of 70 years. Type-1 diabetes is caused due to lack of insulin producing cells in the body which results in hyperglycemia. Associated risk factors such as being overweigth or obese are increasing. Diabetes is an important cause of blindness, kidney failure, lower limb amputation and long term consequences that impact significantly on quality of life. Type-1 Diabetes cannot be prevented with current knowledge. External insulin infusion using insulin pump periodically is the only measure for the Type-1 diabetic patients for keeping blood glucose level to normal level.

In the present study control strategy is proposed for maintaining the blood glucose level as close to normal level. Controller is maintaining the infusion rate of the insulin into the body so as to control the blood glucose level from reaching the dangerous level, while the observer helps in estimating the various states of the model which are impractical to measure or are costly.

2. Controller

2.1 Mathematical Model

Bergman Minimal model is commonly referenced model in literature due to its simplicity and approximately give the dynamic response which close to the normal human body response. It consists of two sets of equations, first represent the glucose response and the second represents about the insulin dynamics in the normal human body.

$$\dot{G}(t) = -p_1[G(t)-G_b]-Z(t)G(t)+D(t) \quad \ldots\ldots\ldots (1a)$$

$$\dot{Z}(t) = -p_2 Z(t) + p_3 [I(t)-I_b] \quad \ldots\ldots\ldots (1b)$$

$$\dot{I}(t) = -\eta [I(t)-I_b] + \gamma[G(t)-h]^t \quad \ldots\ldots\ldots (2)$$

where, $\dot{G}(t)$ is the glucose concentration in the blood plasma in (mg/dl), $\dot{Z}(t)$ is the insulin's effect on the net glucose disappearance, Gb is the basal pre-injection level of glucose in (mg/dl), p_1 is the insulin-independent rate constant of glucose uptake in muscles and liver in (1/min), p_2 is the rate for decrease in tissue glucose uptake ability in (1/min), p_3 is the insulin-dependent increase in glucose up take ability in tissue per unit of insulin concentration above the basal level, $\dot{I}(t)$ is the insulin concentration in plasma at time t in (μ U/ml), Ib is the basal pre-injection level of insulin. Values for parameters will vary from person to person.

IJSER © 2018
http://www.ijser.org

Ḋ(t) shows the rate at which glucose is absorbed to the blood from the intestine, following food intake, considered as disturbance in Type I diabetes mellitus patient and modeled as decaying exponential function as give below.

$$\dot{D}(t) = -B*D(t) \text{ , } B>0 \qquad \ldots\ldots\ldots(3)$$

where, t is in (min) and $D(t)$ is in (mg/dl/min).

2.2 Equilibrium point

The objective is to design the control system such that the system variables in (1)-(3) reach their equilibrium values (i.e., basal values in the present case). For convenience, system of equations introduced in (1)-(3) can be combined and rewritten in its deviation terms with origin as equilibrium point. For this we define

$$[x_1\ x_2\ x_3\ x_4]^T=[x_{10}x_{20}x_{30}x_{40}]^T+[x_{1d}\ x_{2d}\ x_{3d}\ x_{4d}]^T \ \ldots. \ (4)$$

where $[x_{1d}\ x_{2d}\ x_{3d}\ x_{4d}]^T$ are the deviated state about the equilibrium point $[x_{10}\ x_{20}\ x_{30}\ x_{40}]^T$ of the system and $x_1, x_2, x_3\ and\ x_4$ represent Ġ(t), Ż(t), İ(t) and Ḋ(t) respectively. Note that the term ɣ[G(t)−h]† (production of insulin internally) in (2) is removed as it does not exist diabetic patients, u(t) defines the insulin infusion rate and replaces the normal insulin regulation of the body which acts as the control variable. The exogenous infusion of glucose is considered as an additional state variable x_4. The equilibrium points obtained as

$$[x_{10}x_{20}x_{30}x_{40}]^T =[Gb\ 0\ Ib\ 0]^T$$

Hence,

$$\dot{x_{1d}} = -p_1*x_{1d}-x_{2d}\ *(x_{1d} + Gb) + x_{4d} \qquad \ldots.(5)$$
$$\dot{x_{2d}} = -p_2*x_{2d}+ p_3*x_{3d} \qquad .....(6)$$
$$\dot{x_{3d}} = -\eta x_{3d}+U \qquad \ldots.(7)$$
$$\dot{x_{4d}} = -B*x_{4d} \qquad \ldots.(8)$$

2.3 Design

The mathematical model described is divided in three loop structure as follows,

$$\dot{x_{1d}} = -p_1x_{1d}-(x_{1d}+G_b)x_{2d}+x_{4d} \qquad \ldots\ldots\ldots \text{ Loop I}$$
$$\dot{x_{2d}} = -p_2x_{2d}+ p_3x_{3d} \qquad \ldots\ldots\ldots \text{ Loop II}$$
$$\dot{x_{3d}} = -\eta x_{3d}+u(t) \qquad \ldots\ldots\ldots \text{ Loop III}$$

The disturbance due to food intake is the internal dynamics for the system and it can be observed that it is a stable internal dynamics.

In Loop I x_{2d} is considered as input and objective set is to design trajectory of x_{2d} so that $x_{1d} \rightarrow x^*_{1d} = 0$ and for control design purpose x_{2d} acting like input is termed as x^*_{2d} .Defining error between instantaneous blood glucose x_{1d} and desired value of blood glucose $x^*_{1d} = 0$ (implies the fact that $\dot{x}^*_{1d} =0$)as

$$e_1= x_{1d} - x^*_{1d} \qquad \ldots\ldots\ldots(9)$$

The objective in designing x^*_{2d} is to make error $e1$ reach zero as early as possible.

Imposing first-order error dynamics gives

$$\dot{e}_1+ k_1\ e_1= 0 \qquad \ldots\ldots..(10)$$

where $k_1 > 0$ is the design parameter. After appropriate substitution and algebraic rearrangement we are able to compute x^*_{2d} required to meet the objective set for Loop II i.e. to force blood glucose concentration to reach its basal value.

$$x^*_{2d}= \frac{[(-p_1+k1)\ x_{1d}+x_{4d}]}{(x_{1d}+G_b)} \qquad \ldots\ldots(11)$$

Once we know x^*_{2d}which will make sure that glucose level in blood stream reach basal value, our next objective is to make sure x_{2d}follows x^*_{2d}so as to achieve the objective of Loop I. This is achieved in similar manner by consideringx_{3d} in Loop II as input and naming it as $x^*_{3d}$and objective here is to force $x2d$ to follow x^*_{2d}. Defining error between $x_{2d}$and x^*_{2d}as

$$e_2=x_{2d}-x^*_{2d} \qquad \ldots.(12)$$

IJSER © 2018
http://www.ijser.org

The objective in designing x^*_{3d} is to make error e_2 reach zero as early as possible. Imposing first-order error dynamics gives

$$\dot{e}_1 + k_2 * e_2 = 0 \quad \ldots(13)$$

where $k_2 > 0$ is the design parameter. After appropriate substitution and algebraic rearrangement we are able to compute x^*_{3d} required to meet the objective set for Loop II i.e.

$$x^*_{3d} = 1/(p_3[p_2 x_{2d} - k_2(x_{2d} - x^*_{2d})]) \quad \ldots(14)$$

We have assumed that x^*_{2d} is slow varying variable and for small interval of computation of controller it is considered constant (quasi-steady assumption) i.e. $\dot{x}^*_{2d} = 0$ for that duration. Now we know how x_{2d} should follow x^*_{2d} so as to achieve the objective set for Loop II, our next objective is to make sure x_{3d} follows x^*_{3d} so as to achieve the objective of Loop II. This is achieved in similar manner by designing controller for Loop III , which contain our actual controller i.e. rate of insulin to be injected into patients blood. Defining error between x_{3d} and x^*_{3d} as

$$e_3 = x_{3d} - x^*_{3d} \quad \ldots..(15)$$

The objective in designing u_d is to make error e_3 reach zero as early as possible. Imposing first-order error dynamics gives

$$\dot{e}_1 + k_3 * e_2 = 0 \quad \ldots(16)$$

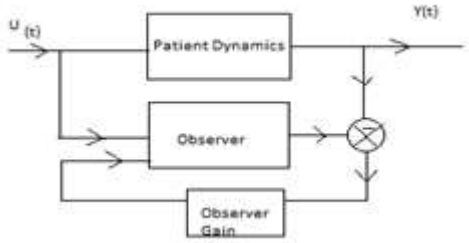
where $k_3 >$ is the design parameter. After appropriate substitution and algebraic rearrangement we are able to compute u_d required to meet the objective set for Loop III i.e.

$$u_d = \eta\, x_{3d} - k_3(x_{3d} - x^*_{3d}) \quad \ldots(17)$$

Similarly, we have assumed that x_{3d} is slow varying variable and for small interval of computation of controller it is considered constant(quasi-steady assumption) i.e. $\dot{x}^*_{3d} = 0$ for that duration. Now we have u_d i.e. rate of insulin to be given to patient is available to make sure that objective set for Loop III is achieved, which ultimately will help to bring down blood glucose level of patient from its initial value to the required basal value.

3. Observer

Luenberger Observer estimates the unknown state variable of the system (in this case patient). Continuous monitoring of the patient parameters is not easily feasible nor economical, so by monitoring the blood glucose level we can estimate the other states by using Observer. This will help in controlling the parameters in the controller and helps in infusion of external insulin. Luenberger observer is used in the linear system to estimates the other state variables using the error between the actual state and the estimated state from the observer. Here the non-linear model has been linearized by the standard method of linearization



Patient Dynamics,

$$x(t) = A*x(t) + B*u(t) \quad \ldots\ldots\ldots(18)$$

$$y(t) = C*x(t) \quad \ldots\ldots\ldots(19)$$

Observer

$$\hat{x}(t) = A*\hat{x}(t) + B*u(t) + L\,(\hat{y}(t) - y(t)) \quad \ldots\ldots\ldots(20)$$

$$\hat{y}(t) = C*\hat{x}(t) \quad \ldots\ldots\ldots(21)$$

where, L is observer gain.

In this case, after linearizing the model we get the following matrices,

$$A = \begin{matrix} -p_1 & -G_b & 0 & 1 \\ 0 & -p_2 & p_3 & 0 \\ 0 & 0 & -\eta & 0 \\ 0 & 0 & 0 & -B \end{matrix}$$

IJSER © 2018
http://www.ijser.org

B=[0 ; 0 ; 1 ; 0]

C=[1 0 0 0]

D=0

By mathematical substitution and solving the equations we get Observer Gain matrix as,

L=[0.6673 ; - 0.00019 ; 15.6542 ; 0.0110]

This value of L matrix helps in the estimating sates of the system approximately.

4. Conclusion

As we can see from fig 1.2, under same disturbance the blood glucose trajectory is different for different patients i.e. G_b. For diabetic type 1 diabetic (blue graph) patient without any external insulin the blood glucose level rises to a dangerous level which may have high risk of death or nervous damage which is normally (hyperglycemia) the case with type-1 diabetic patient. The rest of the trajectories (green, red, cyan) shows that the blood glucose level settle down to the respective basal values by taking proper amount of external insulin which is shown in fig 1.3. In these fig the body insulin level remains unaffected in the body of untreated patient while that of treated patients that dynamically changes. Fig 1.4 shows the insulin infusion rate into the body which is dependent on the amount of glucose present in the blood. From fig 1.5 we can see the effectiveness of the controller as it controls the blood glucose level from different initial points. Fig 1.6 shows the output of the observer i.e, the estimated values of the active plasma insulin which is close to the actual plasma insulin values. Fig1.7 shows the estimated values of blood insulin level by observer which oscillates for some initial time and then follow the actual blood insulin values. This oscillation is due to high error between the actual and the observer initial state values.Fig1.8 show the actual blood glucose level settles to the basal value which is being controlled using Luenberger Observer.

To monitor the actual value of plasma insulin its very complex and costly so by using the observer we can estimate the values very close to the actual values. Moreover it can be observed that by using observer the transient response of the blood glucose concentration has improved while there is no change in the settling time of the blood glucose concentration.

IJSER

References

Journals

[1] Taghreed M. Mohammad, Ridha Mina Qais Kadhum, ShaimaMahmou Mahdi., 2011. Back stepping-Based-PID-Controller Designed for an Artificial Pancreas model. Al-Khwarizmi Engineering Journal, Vol. 7, No. 4, PP 54-60 (2011)

Conferences

[1] Nitin, G, Bhitre and Radhakant Padhi 2011, September. An Adaptive Insulin Infusion Approach for Customized Blood Glucose Regulation of TYPE I Diabetic Patients.In 2011 IEEE International Conference on Control Applications (CCA) Part of 2011 IEEE Multi-Conference on Systems and Control Denver, CO, USA. September 28-30, 2011

Books

[1] Automatic Control Systems by B C. Kuo

Table

TABLE-1

Parameter	Nominal Values	Min Values	Max Values
p_1	0.028	0	0.028
p_2	0.0142	0.01	0.02
p_3	0.0000156	0.000001	0.00002
ɳ	0.2814	0.12	0.3
B	0.05	0.01	0.1

TABLE-2

k_1	0.011764
k_2	0.05
k_3	0.4

Figure

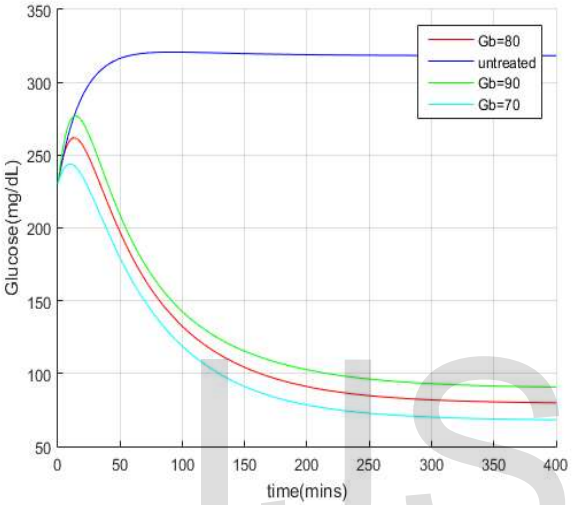


Fig 1.2 Blood Glucose Concentration for different G_b.

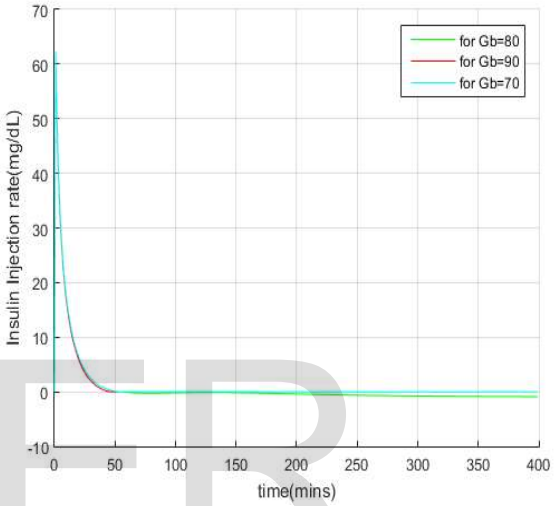


Fig 1.4 Insulin Infusion Rate

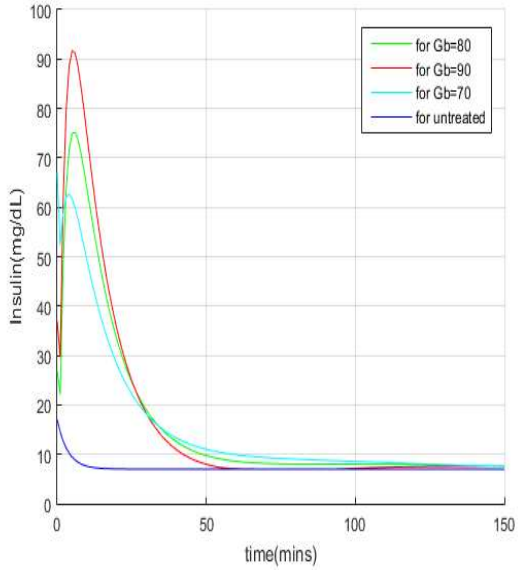


Fig 1.3 Blood Insulin Trajectory for different G_b.

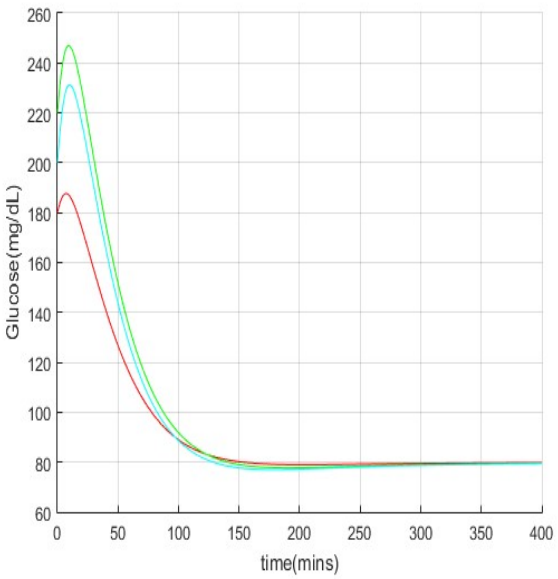


Fig 1.5 Different blood glucose initial values.

IJSER

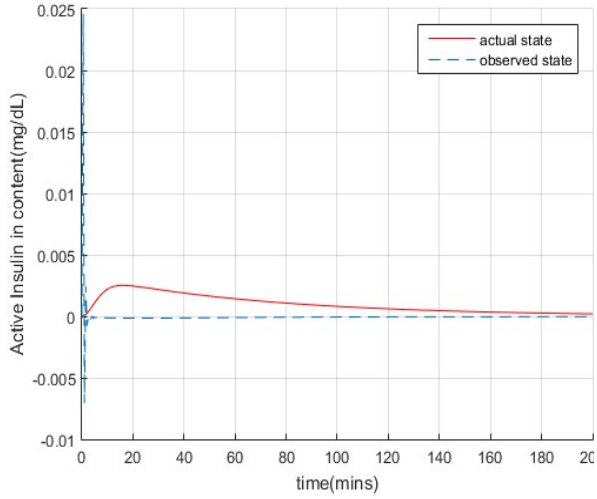


Fig 1.6 Actual state Vs. Observed state of plasma insulin concentration.

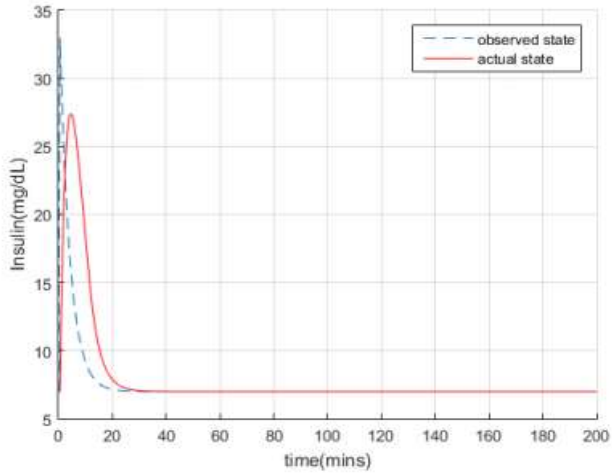


Fig 1.7 Actual State Vs. Observed State of Blood insulin concentration.

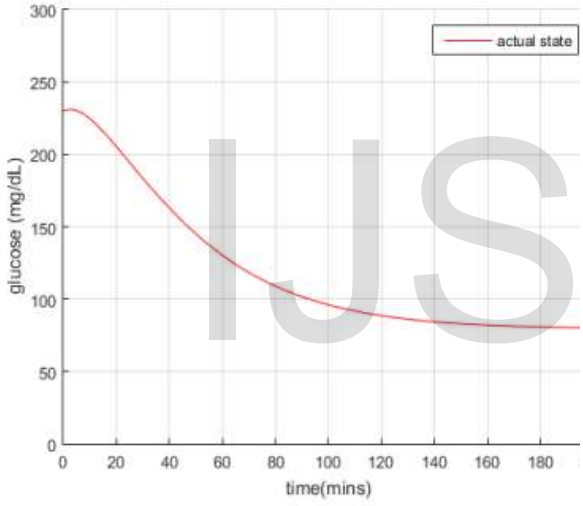


Fig 1.8 Blood Glucose concentration after implementing observer.

IJSER

Authors Biography

Rohit Sharma
B.Tech(2017) in Electrical Engineering from Sardar Patel College of Engineering, India.

N. G. Bhitre
B.E.,M.E.(Electrical)
Assistant Professor in Sardar Patel College of Engineering. Student of Ph.D, department of Aerospace Engineering, Indian Institute of Science, Banglore. 16 years of Teaching Experience.